General Distribution G

Simon Loertscher, Yves Schneider

November 13, 2009

In this file we show which results from the paper "Chain Stores, Consumer Mobility, and Market Structure" generalize to distributions $G(s)$ of setup costs s that are more general than the uniform. We assume that G satisfies the standard monotone hazard rate conditions, i.e. $G(s)/g(s)$ is increasing and $(1-G(s))/g(s)$ decreasing in s. Throughout we take as given that a symmetric SPE exists.[1] As in the paper, we also assume that that the upper bound of the support of G is less than u.

The local's demand in k is $Q_k^l = (2-\alpha)G(\overline{s}_k)+\alpha G(\overline{s}_{-k})$. Notice that $Q_k^l + Q_{-k}^l = 2[G(\overline{s}_k) + G(\overline{s}_{-k})]$. Therefore, the chain's demand can be written as $Q^c = 2[1 - G(\overline{s}_k) + 1 - G(\overline{s}_{-k})]$. Imposing symmetry and substituting $\frac{\partial \overline{s}_k}{\partial p_k} = 2/\alpha$, $\frac{\partial \overline{s}_k}{\partial p_k^l} = -(2-\alpha)/\alpha$ and $\frac{\partial \overline{s}_k}{\partial p_{-k}^l} = -1$ yields the first order conditions

$$p^c = \frac{1-G(\overline{s})}{g(\overline{s})}\frac{\alpha}{2} \tag{1}$$

and

$$p^l = \frac{G(\overline{s})}{g(\overline{s})}\frac{2\alpha}{(2-\alpha)^2+\alpha^2}. \tag{2}$$

Since $\overline{s} = \frac{2}{\alpha}(p^c - p^l)$ because of symmetry, the equilibrium is determined by the fixed point given by

$$\overline{s} = \frac{1-G(\overline{s})}{g(\overline{s})} - \frac{G(\overline{s})}{g(\overline{s})}\frac{4}{(2-\alpha)^2+\alpha^2}. \tag{3}$$

That this fixed point exists and is unique follows from the monotone hazard rates assumptions. The equilibrium profits are

$$\Pi^{c*} = \frac{(1-G(\overline{s}))^2}{g(\overline{s})}\frac{\alpha}{2} \tag{4}$$

[1] Whether or not such undercutting is profitable is hard to determine given the general function G, but can be readily determined for any specific G.

and

$$\Pi^{l*} = \frac{G(\overline{s})^2}{g(\overline{s})} \frac{2\alpha}{(2-\alpha)^2 + \alpha^2}, \tag{5}$$

where $\overline{s}$ is given by (3). It is easy to see that the fixed point $\overline{s}$ decreases in α, which implies that both p^c and Q^c increase in α. Thus, the chain's profit increases in α, and the local's demand decreases. Whether the local's equilibrium price p^l increases in α is not clear without further assumptions on G since there are two opposing effects (the $G(\overline{s})^2/g(\overline{s})$ term decreases and the $\frac{2\alpha}{(2-\alpha)^2+\alpha^2}$ term increases), but it is a necessary condition for the locals' profits to increase with α. Thus, whether or not the locals' profits increase in α is not clear for general distribution G. Similarly, whether or not $\Pi^{c*} \geq 2\Pi^{l*}$ depends on the intricate properties of G. A sufficient condition is $G(x^*) \leq 1/3$, where x^* is the solution to $x = (1 - G(x))/g(x) - G(x)/g(x)$.[2]

[2] The RHS in (3) is less than $(1 - G(x))/g(x) - G(x)/g(x)$ for $\alpha > 0$. Consequently, $\overline{s} < x^*$. Now $\frac{\Pi^{c*}}{\Pi^{l*}} \geq \left(\frac{1-G(\overline{s})}{G(\overline{s})}\right)^2 \frac{1}{2}$, where the RHS is greater than 2 if and only if $G(\overline{s}) \leq 1/3$.